Solved Examples

1. A proton, a deuteron and an α-particle, accelerated through the same potential difference, are projected into a uniform magnetic field with their velocities perpendicular to the field. Find the ratio of the radii of their circular paths.

Sol. When a charged particle enters a magnetic field B with velocity v, the radius of its circular orbit is given by

$$r = \frac{mv}{qB}$$

If E is the kinetic energy, then $v = \sqrt{\frac{2E}{m}}$.

Therefore, $r = \frac{\sqrt{2mE}}{Bq}$

If the particle is initially accelerated through a potential difference V, then

$E = qV$

Therefore, $r = \frac{\sqrt{2mqV}}{Bq} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

If m and e are the mass and charge of a proton, then

mass of deuteron = $2m$

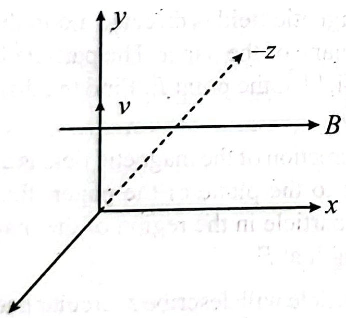
charge of deuteron = e

mass of α– particle = $4m$

charge of α– particle = $2e$

$$\Rightarrow r_p : r_d : r_\alpha = \sqrt{\frac{m}{e}} : \sqrt{\frac{2m}{e}} : \sqrt{\frac{4m}{2e}} = = 1 : \sqrt{2} : \sqrt{2}$$

2. A uniform magnetic field of 30mT exists in the $+x$ direction. A particle of mass 1.67×10^{-27} kg and charge $+1.6 \times 10^{-19}$ C is projected through the field in the $+y$ direction with a speed of 4.8×10^6 m/s.



(*i*) Find the magnitude and direction of the magnetic force on the particle.

(*ii*) Find the force if the particle were negatively charged.

(*iii*) Describe the nature of the path followed by the particle in both cases.

Sol. (*i*) $\vec{F} = q(\vec{v} \times \vec{B})$. Therefore $|\vec{F}| = F = qvB\sin\theta$

$= 30 \times 10^{-3} \times 1.6 \times 10^{-19} \times 4.8 \times 10^6 \times \sin 90°$

$= 2.30 \times 10^{-14}$ N

According to Fleming's left hand rule, the direction of the force is along $-z$ direction.

(*ii*) If the particle were negatively charged, the force will be along $+z$ direction. The magnitude is same as in (*i*)

(*iii*) In both the cases, the path is a circle of radius

$$r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 4.8 \times 10^6}{1.6 \times 10^{-19} \times 30 \times 10^{-3}} = 1.67\text{m}$$

The circle is in the yz plane. In case (*i*) the circular motion is clockwise and in case (*ii*) it is anticlockwise as seen from the $+x$ direction.

3. A current carrying conductor has 8.0×10^{22} free electrons per metre length, having drift velocity 8.0×10^{-5} m/s. If a magnetic field of 0.10T be applied perpendicular to the conductor, then find the force per metre length of the conductor. (Assume cross-section to be unity.)

Sol. Current through the wire is given by

$I = nAev_d$

$= 8 \times 10^{22} \times 1.6 \times 10^{-19} \times 8 \times 10^{-5} = 1.024$A

Force per metre of wire

$= BI = 0.1 \times 1.024 = 0.1$N/m.

4. A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^6 ms^{-1} in the $+x$ direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0, E_z = -102.4$ kVm^{-1} and $B_x = B_z = 0, B_y = 8 \times 10^{-2}$ Wbm^{-2}. The particle enters this region at the origin at time $t = 0$. (*i*) Determine the location (x, y and z coordinate) of the particle at $t = 5 \times 10^6$ s. (*ii*) If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s?

Sol. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be unit vectors along the positive directions of x, y and z axes.

q = charge on the particle = 1.6×10^{-19} C

$\vec{v}$ = velocity of the charged particle = $(1.28 \times 10^6 \text{ ms}^{-1})\,\hat{i}$

$\vec{E}$ = electric field intensity = $(-102.4 \times 10^3 \text{ V m}^{-1})\,\hat{k}$

$\vec{B}$ = magnetic induction of the magnetic field

$= (8 \times 10^{-2} \text{ Wb m}^{-2})$

$\therefore\ \vec{F}_e$ = electric force on the charge

$= q\vec{E} = [(1.6 \times 10^{-19})(-102.4 \times 10^3)\text{N}]\,\hat{k}$

$= (163.84 \times 10^{-16}\text{N})\,(-\hat{k})$

$\vec{F}_m$ = magnetic force on the charge = $q\vec{v} \times \vec{B}$

$= [(1.6 \times 10^{-19})(1.28 \times 10^6)(8 \times 10^{-2})\text{N}]\ (\hat{i} \times \hat{j})$

$= (163.84 \times 10^{-16}\text{ N})\ (\hat{k})$

The two forces $\vec{F}_e$ and $\vec{F}_m$ are along z-axis, equal, opposite and collinear. The net force on the charge is zero and hence the particle does not get deflected and continues to travel along x-axis.

(*i*) At time $t = 5 \times 10^{-6}$ s

$x = (5 \times 10^{-6})(1.28 \times 10^{6}) = 6.4$ m

$\therefore$ Coordinates of the particle = (6.4 m, 0, 0)

(*ii*) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x-z plane, anticlockwise as seen along $+y$ axis.

Now, $\dfrac{mv^2}{r} = qvB$ where r is the radius of the circle

$$\therefore\ r = \frac{mv}{qB} = \frac{(1\times10^{-26})(1.28\times10^{6})}{(1.6\times10^{-19})(8\times10^{-2})} = 1\,\text{m}$$

The length of the arc traced by the particle in

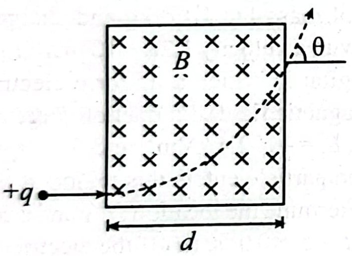
$(7.45 - 5) \times 10^{-6}$ s $= (v)(t)$

$= (1.28 \times 10^{6})(2.45 \times 10^{-6})$

$= 3.136$ m $\approx \dfrac{1}{2}$ circumference $(2\pi r)$

$\therefore$ The particle has the coordinates (6.4 m, 0, 2m) as (x, y, z).

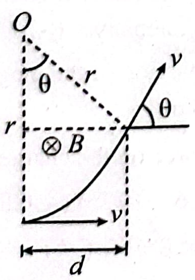
5. A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B. Find the angle of deviation θ of the particle as it comes out of the magnetic field if width d of the region is equal to



(*i*) $\dfrac{mv}{2qB}$ (*ii*) $\dfrac{mv}{qB}$

(*iii*) $\dfrac{2mv}{qB}$

Sol. (*i*) The radius of the circular orbit is given by $r = \dfrac{mv}{qB}$

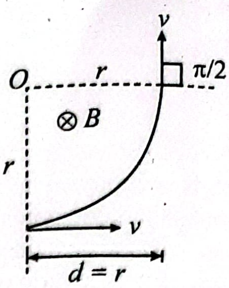


The angle of deviation is given by

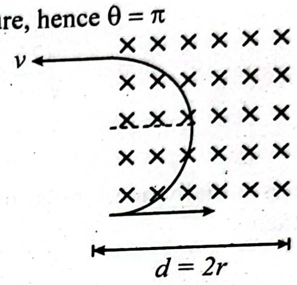
$$\sin\theta = \frac{d}{r} = \frac{1}{2}\left(\because d = \frac{mv}{2qB} = \frac{r}{2}\right)$$

or $\theta = \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

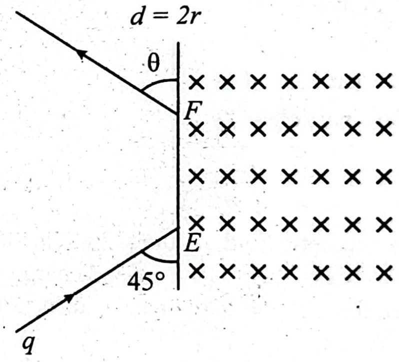
(*ii*) When $d = \dfrac{mv}{qB} = r$, the charged particle deviates through an angle of $\dfrac{\pi}{2}$ as shown in the figure, hence $\theta = \dfrac{\pi}{2}$



(*iii*) When $d = 2\dfrac{mv}{qB} = 2r$ the charged particle completes one semi-circle and deviates through π, as shown in the figure, hence $\theta = \pi$



6. A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$ C enters a region of uniform magnetic field of strength 1 T as shown in the figure. The speed of the particle is 10^7 ms^{-1}.



(*i*) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ.

(*ii*) If the direction of the magnetic field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.

Sol. (*i*) The particle will describe a circular path. Since the path is symmetrical about the entrance and the exit point the angle $\theta = 45°$.

Draw normals to the path at E and F and let them meet at O, Then O is the centre of the circular path. The angle $EOF = 90°$, therefore, the path is a quarter of a circle.

Centripetal force $= qvB = m\omega^2 r$

$q(\omega r) = m\omega^2 r$

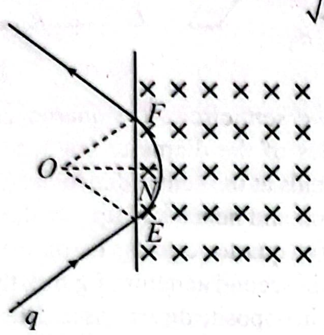
$\therefore\ \omega = \dfrac{qB}{m} \Rightarrow T = \dfrac{2\pi m}{qB}$

$$\Rightarrow T = \frac{2\times\pi\times 1.6\times 10^{-27}}{1.6\times 10^{-19}\times 1} = 6.28\times 10^{-8}$$

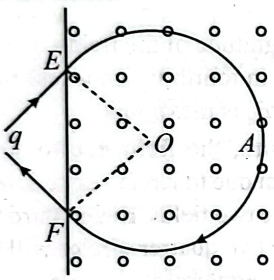
Since $v = \omega r, r = \frac{vT}{2\pi} = \frac{10^7\times 6.28\times 10^{-8}}{2\pi} = 0.1$ m

Thus $OE = r = 0.1$ m

$\therefore\ EF = 2OE \cos 45° = 2\times 0.1\times \frac{1}{\sqrt{2}} = 0.1414$ m



(*ii*) When the field is reversed, the point of emergence F is at the same distance on the other side of E. The particle now describes three quarters of a circle of the same radius. Now the time spent by the particle in the magnetic field is the time required to describe three-fourths of the circle.



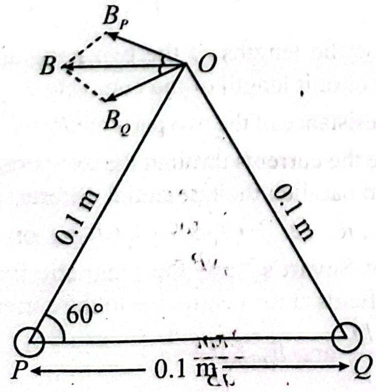
$\therefore\quad t = (3/4)\times 6.28\times 10^{-8} = 4.71\times 10^{-8}$ s

7. Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 ampere each. Find the magnetic field at a point distant 0.1 m from both wires in the two cases when the currents are in the (*i*) same and (*ii*) opposite directions.

Sol. (*i*) Currents in the same direction:

Let P and Q be the two wires carrying currents in the same direction, out of the paper. The magnetic field at O due to the wire P

$$B_P = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times 10^{-7}\times 10}{2\pi\times 0.1} = 2\times 10^{-5} T,$$



directed perpendicular to OP as shown in above figure. Magnetic field at O due to the wire Q is $B_Q = 2\times 10^{-5}$ T, directed perpendicular to OQ, as show in the above figure.

Resultant field at O,

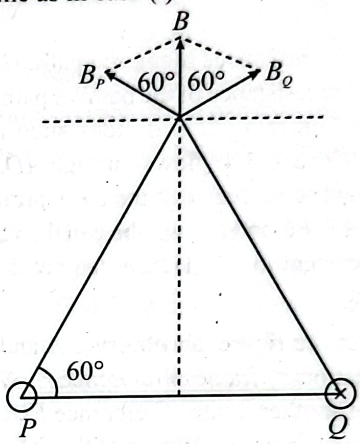
$B = 2B_P \cos 30°\qquad$ (as $B_P = B_Q$)

$= 2\times 2\times 10^{-5}\times(\sqrt{3}/2)$

$= 3.46\times 10^{-5}$ T, parallel to QP

(*ii*) Currents in opposite directions:

Let the current in P be out of the paper and that in Q be into the paper. The direction of B_P is as in case (*i*). The direction B_Q is shown in figure below and the magnitude is same as in case (*i*).



Resultant field $B = 2B_P \cos 60°$

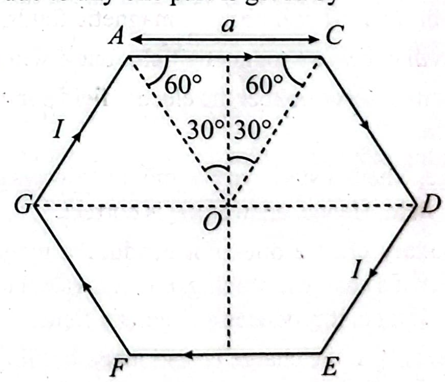
$= 2\times 2\times 10^{-5}\times(1/2)$

$= 2\times 10^{-5}$ T, perpendicular to PQ

8. A current carrying network is in the shape of a regular hexagon of side a. If the current through the field network is I, find the magnetic field at the centre of the hexagon.

Sol. The magnetic field at O has the same magnitude and direction due to all the six straight parts.

Field due to any one part is given by



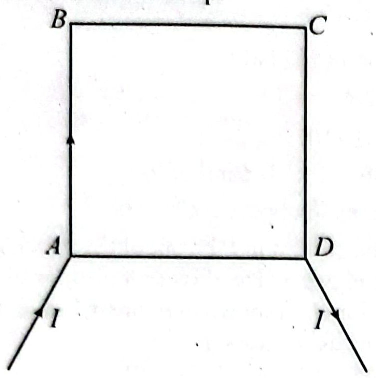
$$B' = \frac{\mu_0}{4\pi(a/2)\tan 60°}[\sin 30° + \sin 30°]$$

$$= \frac{\mu_0}{2\pi a\sqrt{3}}\left[2\times\frac{1}{2}\right] = \frac{\mu_0}{2\sqrt{3}\pi a},$$ directed into the paper.

Field due to the whole hexagon is

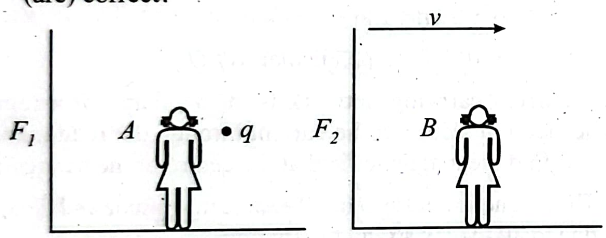
$$B = 6B' = \frac{\sqrt{3}\mu_0}{\pi a},$$ directed into the paper.

9. A uniform conducting wire forms a square loop $ABCD$. A current enters the loop at A and leaves at D. Find the magnetic field at the centre of the loop.



Sol. Obviously, the resistance along the path $AB + BC + CD$ is three times the resistance of the parallel path AD. Therefore, the current I entering at A divides such that 1/4th flows through $ABCD$ and 3/4th flows through AD. The combined magnetic field at the centre of the loop, produced due to the currents in AB, BC and CD will be equal and opposite to that due to the current in AD. Hence, the net field at the centre will be zero.

10. As shown in the figure, an observer A and a charge q are fixed in a stationary frame of reference F_1. Another observer B is fixed in another frame of reference F_2, which is moving with respect to frame F_1. Which of the following choices is (are) correct?



(*a*) Both A and B will observe electric field.

(*b*) Both A and B will observe magnetic fields.

(*c*) B will observe a magnetic field, but A will not.

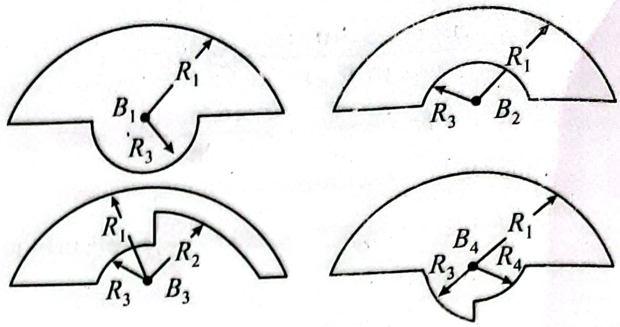
(*d*) B will observe neither the electric field nor the magnetic field.

Sol. A charge, whether stationary or moving always produces an electric field. Hence, answer (*a*) is correct.

A stationary charge does not produce a magnetic field. However, if a charge is moving, it is equivalent to an electric current. Hence, it produces a magnetic field.

For observer A, the charge is stationary, but for observer B it is moving (in opposite direction to the frame F_2). Hence, answer (*c*) is also correct.

11. In the four loops shown in the figure, all curved sections are either semicircles or quarter circles of radii R_1, R_2, R_3 and R_4 such that $R_1 > R_2 > R_3 > R_4$. All the four loops carry the same current. If B_1, B_2, B_3 and B_4 are the magnitude of the magnetic fields at the centres of the loops, then rank them according to magnitude, greatest fir t.



Sol. When the two semicircles (or quarter circles) are on the opposite sides of the diameter (as in the first and fourth figure) the fields at the centre due to the two parts are in the same direction and hence add up. On the other hand, if the semicircles (or quarter circles) lie on the same side of the diameter (as in second and third figure), the fields due to the two parts are in opposite directions and hence get subtracted. Therefore, fields B_1 and B_4 will have greater magnitudes than the fields B_2 and B_3.

As per Biot-Savart law, the magnitude of the magnetic field at the centre of a curved current carrying elements is

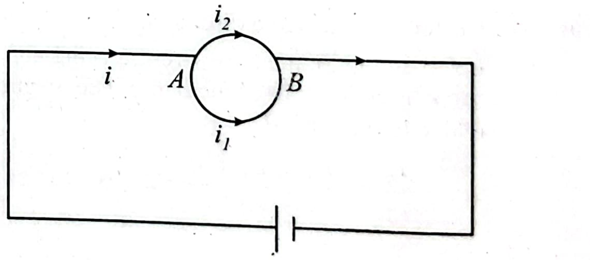
$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{IdL}{R^2}$$

That is the magnitude of the fields is inversely proportional to R^2. Since R_4 in fourth figure is less than R_3 in first figure, $B_4 > B_1$. Thus, B_4 is maximum.

In second figure, the field due to smaller semicircle is greater than that due to larger semicircle. The net field is the difference of the two fields. But, in third figure, the combined field due to the two quarter-circles will be less than that due to the smaller semicircle in second figure. Therefore, the field $B_3 < B_2$

Thus, B_3 is the minimum, and $B_4 > B_1 > B_2 > B_3$

12. Any two points A and B on a uniform circular conductor are connected to a cell. Show that the total induction of the magnetic field at the centre is zero.



Sol. Let l_1, l_2 be the lengths of the two parts and let σ be the resistance of unit length of the conductor.

Then the resistance of the two parts are $R_1 = l_1\sigma$ and $R_2 = l_2\sigma$.

Let i_1, i_2 be the currents through the two parts. Since the two parts are in parallel, their potential differences are equal.

$\therefore\ i_1 R_1 = i_2 R_2 \quad \therefore\ i_1(l_1\sigma) = i_2(l_2\sigma)$ or $i_1 l_1 = i_2 l_2$

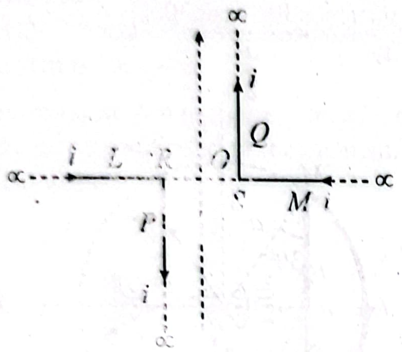
From Biot-Savart's Law, the magnetic induction of the magnetic fields at the centre due to the currents i_1 and i_2 are

and $B_1 \propto \frac{l_1 i_1}{r^2}$ and $B_2 \propto \frac{l_2 i_2}{r^2}$.

The directions of these fields are opposite to each other. Hence the total magnetic induction at the centre is

$$B = (B_1 - B_2) \propto \left(\frac{l_1 i_1}{r^2} - \frac{l_2 i_2}{r^2}\right) = 0 \text{ because } l_1 i_1 = l_2 i_2 .$$

13. A pair of stationary and infinitely long bent wires are placed in the XY plane as shown in figure. The wires carry currents of $i = 10$ amperes each as shown. The segments P and Q are parallel to the Y axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O.



Sol. $i = 10$ A, $OS = OR = 0.02$ m $= r$

The magnetic induction at O due to LR and MS is zero (the point is on the line of the current)

Magnetic induction at O due to QS,

$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i}{r}$ (outward the paper)

Magnetic induction at O due to PR

$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{i}{r}$ (outward of the paper)

Then net induction at O

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2i}{r} = (10^{-7})(2)\left(\frac{10}{0.02}\right) = 10^{-4}\text{T}$$

outward of the paper

14. What is the force between two equal charges moving parallel to each other with the same velocity?

Sol. In figure, the repulsive force between the charges is

$$F_E = \frac{kq^2}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$$

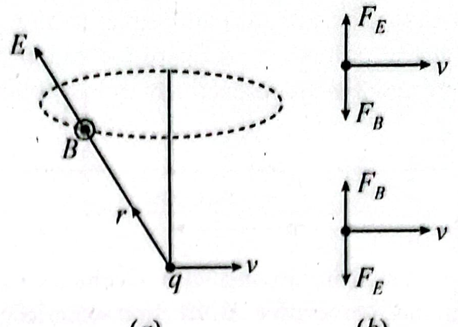
The magnitude of the magnetic force exerted by one charge on the other is $\vec{F}_B = q\vec{v} \times \vec{B}$

$$F_B = (qv)\left(\frac{\mu_0}{4\pi}\frac{qv}{d^2}\right) = \frac{\mu_0}{4\pi}\frac{q^2 v^2}{d^2}$$

This force is attractive.

The net force between the charges is

$$F = F_E - F_B = \frac{q^2}{4\pi d^2}\left[\frac{1}{\varepsilon_0} - \mu_0 v^2\right]$$

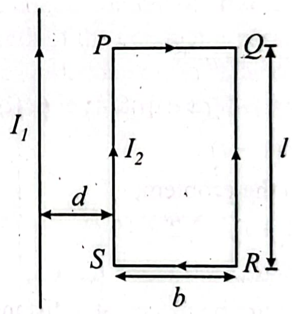


(a) (b)

The magnetic field produced by a charge q moving at velocity v is shown in (Figure a). Two charges moving side-by side with the same velocity is shown in (Figure b). The net force between them is less than when they are at rest i.e.(when there is no magnetic force)

The net force on each of the particles moving with the same velocity is less than that when they are at rest.

15. The figure shows a rectangular current carrying loop $PQRS$ placed with its longer side parallel to a long straight current carrying wire. If $I_1 = 20$ A, $I_2 = 16$ A, $l = 15$ cm, $b = 6$ cm and $d = 4$ cm, find the magnitude and direction of the force experienced by the current loop due to the long wire. How will the force change if the direction of current in the loop is reversed?



Sol. Force on the side PS due to the long straight wire.

$$F_{PS} = \frac{\mu_0 I_1 I_2}{2\pi d}\ell = \frac{2\times10^{-7}\times20\times16\times0.15}{0.04}$$

$= 2.4 \times 10^{-4}$ N, towards left

Force on the side QR due to the long straight wire

$$F_{QR} = \frac{2\times10^{-7}\times20\times16\times0.15}{(0.04+0.06)}$$

$= 0.96 \times 10^{-4}$ N, towards right

The forces on the sides PQ and RS are equal and opposite and so they cancel out.

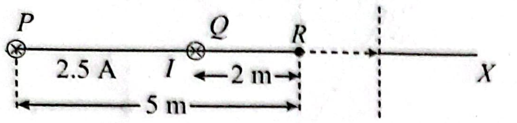
Net force on the loop:

$F = F_{PS} - F_{QR} = 1.44 \times 10^{-4}$ N, towards left.

If the direction of current in the loop is reversed, the magnitude of the force remains the same but its direction is reversed.

16. Two long parallel wires carrying current 2.5 ampere and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively

such that they are perpendicular to the plane of the paper. The points P and Q are located at a distance of 5 metre and 2 metre respectively from a collinear point R



(*i*) An electron moving with a velocity of 4×10^5 m/s along the positive X-direction experiences a force of magnitude 3.2×10^{-20} N at the point R. Find the value of current I.

(*ii*) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 ampere may be placed so that the magnetic induction at R is zero.

Sol. The magnetic field induction at point R due to current in wires P and Q is given by

$$B = B_1 + B_2 = \left(\frac{\mu_0 I_1}{2\pi d_1} + \frac{\mu_0 I_2}{2\pi d_2}\right)$$

$$= 2\times10^{-7}\left(\frac{2.5}{5}+\frac{I}{2}\right) \text{ along negative } Y\text{-axis}$$

$$= -10^{-7}(1+I)\hat{j}\ (\because I_2 = I)$$

where $\frac{\mu_0}{2\pi} = 2\times10^{-7}$ and $\hat{j}$ being the unit vector along Y-axis.

(*i*) We know that $F = qvB$

$= (-1.6 \times 10^{-19}) \times \{(4 \times 10^5\ \hat{i}) \times \{-10^{-7}(1+I)\ \hat{j}\}$

$= 6.4 \times 10^{-21}\ (1 + I)$

According to the problem,

$6.4 \times 10^{-21}\ (1 + I) = 3.2 \times 10^{-20}$

$\therefore\ I = 4$ ampere

(*ii*) Let the third wire be placed at a distance x on the left of R. Suppose the wire carries a current upwards perpendicular to the plane of paper. Now

$$B = B_1 + B_2 + B_3$$

$$= \frac{\mu_0}{2\pi}\left(\frac{I_1}{d_1}+\frac{I_2}{d_2}+\frac{I_3}{x}\right) = 0$$

($\therefore$ Directions of I_1 and I_2 are along negative Z-direction and direction of I_3 is along positive Z-direction)

or $\quad B = \frac{2.5}{2}+\frac{4}{2}-\frac{2.5}{x} = 0$

$\therefore\quad x = 1$ metre

Note: If the direction of current in the third wire will be inward (negative Z-direction), then

$$B = \frac{2.5}{5}+\frac{4}{2}-\frac{2.5}{x} = 0$$

$\therefore\ x = -1$ metre

Hence the third wire may be placed at ±1 metre from R along X-axis.

17. A wire loop carrying a current I is placed in the x-y plane as shown in the figure.

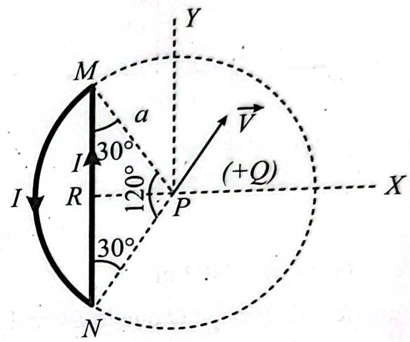
(*i*) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\vec{V}$ along NP (see figure), find its instantaneous acceleration.

(*ii*) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.

Sol. (*i*) $PR = \frac{a}{2}$

$\vec{B}_1$ = magnetic induction at P due to current I in chord NM

$$= \frac{\mu_0}{4\pi}\frac{I(\cos 30° + \cos 30°)}{\frac{a}{2}}(-\hat{k})$$



$\frac{\mu_0 I}{4\pi}\frac{2\sqrt{3}}{a}(-\hat{k}) = \frac{\mu_0 I}{4\pi}\frac{2\sqrt{3}}{a}$, directed into the plane of figure.

$\vec{B}_2$ = the magnitude induction at P due to arc MN

$\frac{\mu_0}{4\pi}\frac{I}{a}\frac{2\pi}{3}$ outward from the plane of figure $= \frac{\mu_0}{4\pi}\frac{I}{a}\frac{2\pi}{3}\hat{k}$

$\vec{B}$ = net magnetic induction at P due to the loop

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[2\sqrt{3}-\frac{2\pi}{3}\right] \text{ directed into the plane of the figure}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}(1.369)(-\hat{k}) \text{ tesla}$$

$\vec{V}$ = velocity of $+Q$ charge at P

$\vec{F} = Q\vec{V}\times\vec{B}$

$$(\vec{F}) = (Q)\,(V)\left[\frac{\mu_0}{4\pi}\frac{I}{a}(1.369)\right]$$

($\vec{V}$ and $\vec{B}$ are perpendicular to each other) perpendicular to NP in the XY plane towards the loop

$\therefore\ \vec{a}$ = acceleration of the charge at

$$P = \frac{\vec{F}}{m} = \frac{QV}{m}\left[1.369\frac{\mu_0}{4\pi}\frac{I}{a}\right]$$

perpendicular to NP in XY plane towards the loop.

(*ii*) When a uniform magnetic field is introduced along X-axis, the loop in the XY plane will be parallel to the magnetic induction and hence experiences zero net force but experiences only a torque $\vec{\tau}$ given by

$\vec{\tau} = I\vec{A}\times\vec{B}$

$\vec{A}$ = area vector of the coil

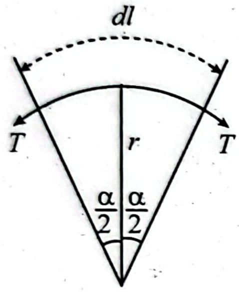
$$= \left[\frac{\pi a^2}{3} - \frac{1}{2}(2a\cos 30°)\frac{a}{2}\right]\hat{k},\ \vec{B} = B\hat{i}$$

$$\therefore\ |\vec{\tau}| = (I)(a^2)\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)(B)(\hat{k}\times\hat{i}),\ \vec{\tau} \text{ is along } Y\text{-axis}$$

$$\therefore\ \vec{\tau} = Ia^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)B\hat{j}$$

18. A loop of flexible conducting wire of length l lies in a magnetic field of B perpendicular to the plane of the loop. Show that when a current is passed through the loop, it opens into a circle. Also calculate the tension developed in the wire if the current is i ampere.

Sol. The situation is shown in figure. Force on every element is $iBdl$. This is perpendicular to the element. Hence the loop opens into a circle.



Now, $2T\sin\frac{\alpha}{2} = iBdl$

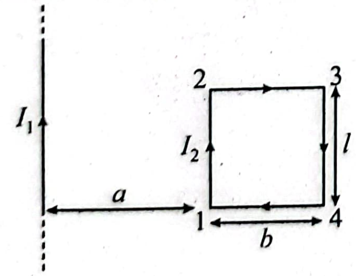
or $\quad T\alpha = iBdl\left(\because \sin\frac{\alpha}{2} = \frac{\alpha}{2},\ \text{for small angle}\right)$

$$\therefore\quad T = \frac{iBdl}{\alpha} = iBr\left(\because \frac{dl}{r} = \alpha\right) = \frac{iBl}{2\pi}(\because 2\pi r = l)$$

19. A rectangular loop of length l and width b carrying a current I_2 is placed in the neighbourhood of a long straight wire carrying current I_1 and shown in figure.

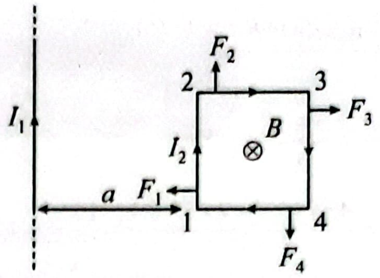
(*i*) Find the net force acting on the loop

(*ii*) Find the work done to increase the spacing between the loop and the wire from a to $2a$.



Sol. (*i*) The force acting on each side of the loop are shown in figure. Obviously, the magnitude of F_2 is equal to that of F_4. Thus, there is no net vertical force on the loop. The magnitude of F_1 and F_3 are

$$F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi a}\ F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi(a+b)}$$



The net force on the loop is horizontal and it is attractive.

$$F_{\text{net}} = F_1 - F_2 = \left[\frac{1}{a} - \frac{1}{a+b}\right]$$

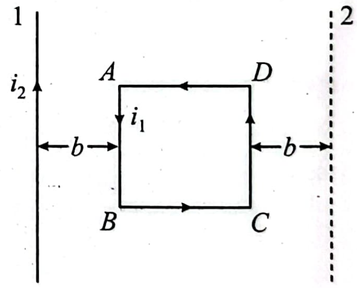
(*ii*) If F is the instantaneous force acting on the loop when its separation from the wire is x, then the work done to increase the spacing from a to $2a$ is

$$W = \int_a^{2a} Fdx \text{ where } F = \frac{\mu_0 I_1 I_2 l}{2\pi}\left[\frac{1}{x} - \frac{1}{x+b}\right]$$

$$\text{Thus, } W = \frac{\mu_0 I_1 I_2 l}{2\pi}\left[\int_a^{2a}\frac{1}{x}dx - \int_a^{2a}\frac{1}{x+b}dx\right]$$

$$\text{or } W = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left|\frac{2(a+b)}{2a+b}\right|$$

20. A square frame $ABCD$ of side $a = 10$ cm with a steady current $i = 2$A is near a long straight conductor carrying current $i_2 = 5$A. The conductor and the frame are in the same plane, with the length of the conductor parallel to the side AB and CD. Find the work done to move the straight conductor from position 1 to position 2. Given that $b = 5$ cm.



Sol. Let us first find the mutual potential energy between a long conductor and a short parallel conductor of length a. When they are apart by z the force of attraction between them is $(\mu_0 i_1 i_2 a)/2\pi z$. Then the work done by an agent to move it from $z = l$ (a large distance) to $z = x$ is

$$\int_l^x -\frac{\mu_0 i_1 i_2 a dz}{2\pi z} = -\left[\frac{\mu_0 i_1 i_2 a \ln x}{2\pi} + C \text{ (a constant)}\right]$$

By definition, this is the mutual potential energy. We can now write the mutual potential energy of the loop $ABCD$ and the conductor in positions 1 and 2. Remembering that repulsion corresponds to positive potential and attraction to negative potential

$$U_i = \left[\frac{\mu_0 i_1 i_2 a \ln b}{2\pi} + C\right] - \left[\frac{\mu_0 i_1 i_2 a}{2\pi}\ln(a+b) + C\right]$$

$$= \frac{\mu_0 i_1 i_2 a}{2\pi}\ln\frac{b}{a+b}$$

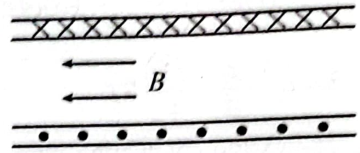
$$U_f = -\left[\frac{\mu_0 i_1 i_2 a \ln b}{2\pi} + C\right] + \left[\frac{\mu_0 i_1 i_2 a}{2\pi}\ln(a+b) + C\right]$$

$$= \frac{\mu_0 i_1 i_2 a}{2\pi}\ln\frac{a+b}{b}$$

$$\therefore \quad W = \frac{4\pi\times10^{-7}\times2\times5\times0.1}{\pi}\ln\frac{15}{5} = 0.44\times10^{-7}\,J$$

21. **Find the magnitude and direction of the magnetic induction due to two infinite planes carrying current of linear density j and $-j$, one perpendicular into and the other out of the paper.**

Sol. At any point outside the sheets, the fields due to the two sheets are equal and opposite and hence add upto zero. At any point between the planes, the two fields are equal and in the same direction.



$\therefore \quad B = 0$ outside and $B = \mu_0 j$ inside.